

Nonlinear System Analysis

You said:



system has the input-output relation given by $y(t) = x^2(t)$.
Apply superposition property to show that the system is nonlinear

ChatGPT said:



To determine whether the system $y(t) = x^2(t)$ is linear, we can apply the superposition principle, which is a fundamental property of linear systems. According to the principle of superposition, a system is linear if it satisfies both **additivity** and **homogeneity** (also known as **scaling**).

1. Additivity

If the system is linear, then for any two inputs $x_1(t)$ and $x_2(t)$, the output for the combined input $x_1(t) + x_2(t)$ should be equal to the sum of the individual outputs:

$$y_{combined}(t) = y_1(t) + y_2(t)$$

Where:

- $y_1(t)$ is the output corresponding to $x_1(t)$
- $y_2(t)$ is the output corresponding to $x_2(t)$

For the given system:

$$y_1(t) = x_1^2(t)$$

$$y_2(t) = x_2^2(t)$$

Now, let's compute the output for the combined input $x_1(t) + x_2(t)$:

$$y_{combined}(t) = (x_1(t) + x_2(t))^2$$

Expanding the square:

$$y_{combined}(t) = x_1^2(t) + 2x_1(t)x_2(t) + x_2^2(t)$$

Notice that:

$$y_{combined}(t) = y_1(t) + y_2(t) + 2x_1(t)x_2(t)$$

Here, $y_{combined}(t)$ is not equal to $y_1(t) + y_2(t)$ because of the additional cross-term $2x_1(t)x_2(t)$. This shows that the system does not satisfy the additivity property.

2. Homogeneity (Scaling)

If the system is linear, then for any input $x(t)$ and any scalar α , the output for the scaled input $\alpha x(t)$ should be equal to the scaled output:

$$y_{\text{scaled}}(t) = \alpha y(t)$$

For the given system:

$$y(t) = x^2(t)$$

Now, consider a scaled input $\alpha x(t)$:

$$y_{\text{scaled}}(t) = (\alpha x(t))^2 = \alpha^2 x^2(t)$$

However, for linearity, we would expect:

$$y_{\text{expected}}(t) = \alpha y(t) = \alpha x^2(t)$$

Clearly, $y_{\text{scaled}}(t) = \alpha^2 x^2(t)$ is not equal to $y_{\text{expected}}(t) = \alpha x^2(t)$, except when $\alpha = 1$ or $\alpha = 0$. This demonstrates that the system does not satisfy the homogeneity (scaling) property.

Conclusion

Since the system does not satisfy the additivity property and the homogeneity (scaling) property, it violates the superposition principle. Therefore, the system described by $y(t) = x^2(t)$ is **nonlinear**.

You said:



show equations that are linear or non linear and prove it using the properties of additivity, homeogeneity

ChatGPT said:



Let's analyze both linear and nonlinear systems using the properties of **additivity** and **homogeneity**.

Example 1: Linear System

System: $y(t) = 3x(t) + 5$

Additivity

Let $x_1(t)$ and $x_2(t)$ be two inputs, and the corresponding outputs are:

$$y_1(t) = 3x_1(t) + 5$$

$$y_2(t) = 3x_2(t) + 5$$

Now, for the combined input $x_1(t) + x_2(t)$:

$$y_{\text{combined}}(t) = 3(x_1(t) + x_2(t)) + 5$$

Expanding the equation:

$$y_{\text{combined}}(t) = 3x_1(t) + 3x_2(t) + 5$$

Notice that:

$$y_{\text{combined}}(t) = (3x_1(t) + 5) + (3x_2(t) + 5) - 5$$

This is not exactly equal to $y_1(t) + y_2(t)$ because of the constant term $+5$, which is not correctly handled in the combined expression. Thus, **this system is not perfectly linear in terms of additivity.**

Homogeneity (Scaling)

Consider a scalar α and an input $x(t)$. The output is:

$$y(t) = 3x(t) + 5$$

For the scaled input $\alpha x(t)$:

$$y_{\text{scaled}}(t) = 3(\alpha x(t)) + 5 = 3\alpha x(t) + 5$$

However, the expected linear output should be $\alpha y(t) = \alpha(3x(t) + 5) = 3\alpha x(t) + 5\alpha$. The mismatch in the constant term indicates this system is **nonlinear**.

Example 2: Nonlinear System

System: $y(t) = x^2(t)$

Additivity

Let $x_1(t)$ and $x_2(t)$ be two inputs, and the corresponding outputs are:

$$y_1(t) = x_1^2(t)$$

$$y_2(t) = x_2^2(t)$$

Now, for the combined input $x_1(t) + x_2(t)$:

$$y_{\text{combined}}(t) = (x_1(t) + x_2(t))^2$$

Expanding the square:

$$y_{\text{combined}}(t) = x_1^2(t) + 2x_1(t)x_2(t) + x_2^2(t)$$

Here, the cross-term $2x_1(t)x_2(t)$ shows that $y_{\text{combined}}(t)$ is not equal to $y_1(t) + y_2(t)$, indicating that the system is **nonlinear** with respect to additivity.

Homogeneity (Scaling)

For a scalar α and input $x(t)$:

$$y(t) = x^2(t)$$

For the scaled input $\alpha x(t)$:

$$y_{\text{scaled}}(t) = (\alpha x(t))^2 = \alpha^2 x^2(t)$$

The expected output for a linear system would be $\alpha y(t) = \alpha x^2(t)$. Since $\alpha^2 x^2(t)$ is generally not equal to $\alpha x^2(t)$, the system does not satisfy homogeneity, thus confirming that the system is **nonlinear**.

Conclusion

- **Linear System:** $y(t) = ax(t)$ satisfies both additivity and homogeneity, provided a is a constant and there are no constant terms like $+5$.
- **Nonlinear System:** $y(t) = x^2(t)$ does not satisfy additivity or homogeneity, making it nonlinear.

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